☺ 1.3 – A First Look at Limits ☺

Objectives:

1. Explore long-run values
2. Begin to understand the concept of limit
3. Investigate shifted geometric sequences

Read the 1.3 introduction on page 48. **Note the types of sequences that do not simply get larger and larger or smaller and smaller**

Investigation – Doses of Medicine

Our kidneys continuously filter our blood, removing impurities. Doctors take this into account when prescribing the dosage and frequency of medicine.

In this investigation, you will simulate what happens in the body when a patient takes medicine.

To represent the blood in a patient's body, use a bowl containing 1 liter of water. Start with 16 milliliters of tinted liquid to represent a dose of medicine in the blood.

Step 1: Suppose a patient's kidneys filter out 25% of this medicine each day. To simulate this, remove ¼ or 25 mL of the mixture from the bowl and replace it with 250 mL of clear water to represent the filtered blood. Make a table to record the amount of medicine in the blood after several days:

Day	Amount of medicine (mL)
0	16
1	12
2	9
3	6.75

Step 2: Write a recursive formula that generates the sequence in your table above.

$u_0 = 16$

$u_n = (1-.25)u_{n-1}$ where $n \geq 1$

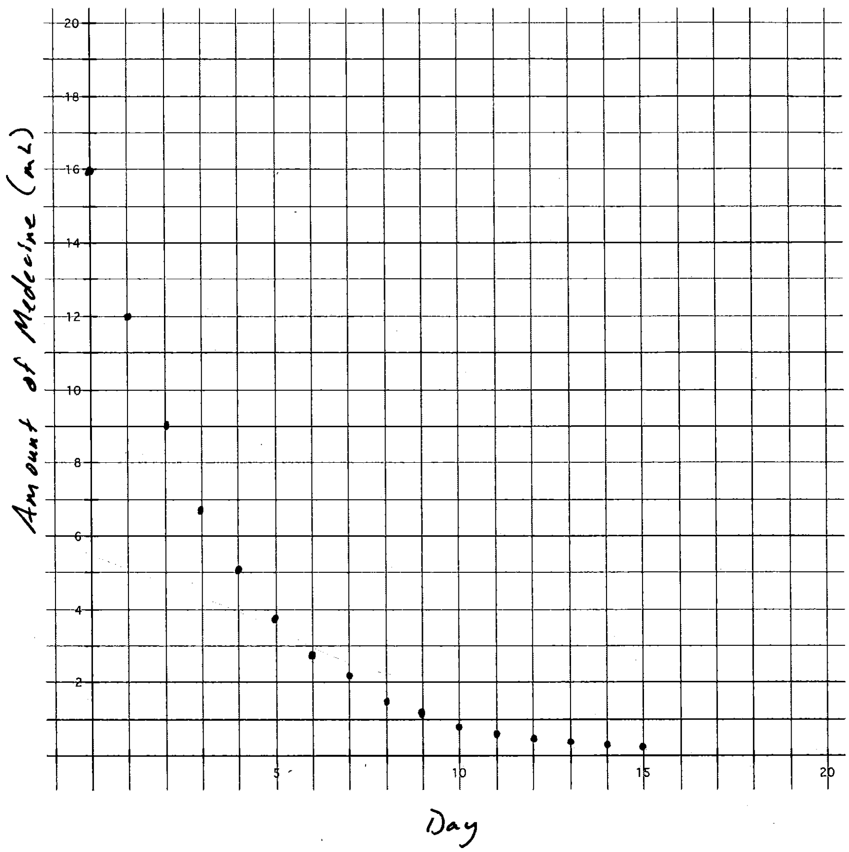
Step 3: How many days will pass before there is less than 1 mL of medicine in the blood?

10 days (.901)

Step 4: Is the medicine ever completely removed from the blood? Why or why not?

No, there is only a percentage removed. It will get smaller and smaller but will not reach zero.

Step 5: Sketch the graph up until the 15th day and describe what happens in the long-run:



The graph decreases quickly at first but the rate of decrease slows as time passes until the graph begins to approach zero.

A single dose of medicine is often not enough to treat a patient's condition. Patients often need to take multiple doses to produce and maintain a high enough level of medicine in the body. Now we will modify the simulation to look at what happens when a patient takes medicine daily over a period of time.

Step 6: Start again with 1 L of water, representing the blood, except for the 16 mL of tinted liquid (medicine). Each day 250 mL of liquid is removed and replaced with 234 mL of clear water (filtered blood) and 16 mL of tinted liquid (medicine). Complete the table below to show how much medicine is in the blood after a number of days:

Day	Amount of medicine (mL)
0	16
1	28
2	37
3	43.75

Step 7: Write a recursive routine to generate the sequence in your table above:

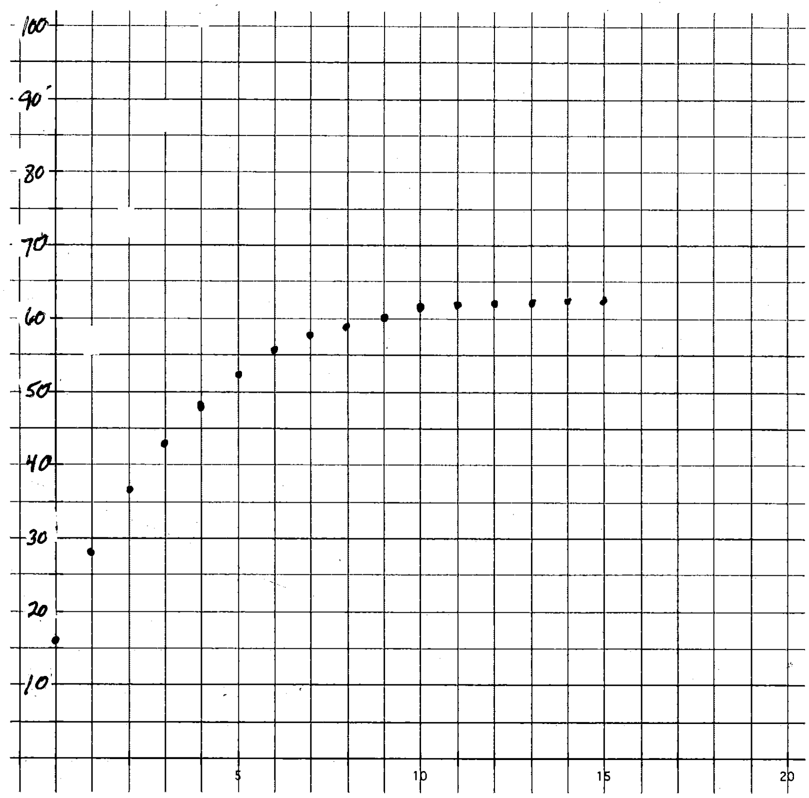
$u_0 = 16$

$u_n = .75u_{n-1} + 16$

Step 8: Do the contents of the bowl ever turn into pure medicine? Why or why not?

No, there is always pure water added at each step.

Step 9: Sketch a graph through the 15th day and explain what happens to the level of medicine in the blood after many days.



Approaches 64

Limit: A long-run value that a sequence or function approaches.

Shifted Geometric Series: A geometric series that includes an added term in the recursive rule.

Example 1: Antonio and Deanna are working at the community pool for the summer. They need to provide a shock treatment of 450 grams of dry chlorine to prevent the growth of algae in the pool. Then they add 45 g of chlorine each day after the initial treatment. Each day, the sun burns off 15% of the chlorine.

a. Write a recursive routine to represent the situation described above.

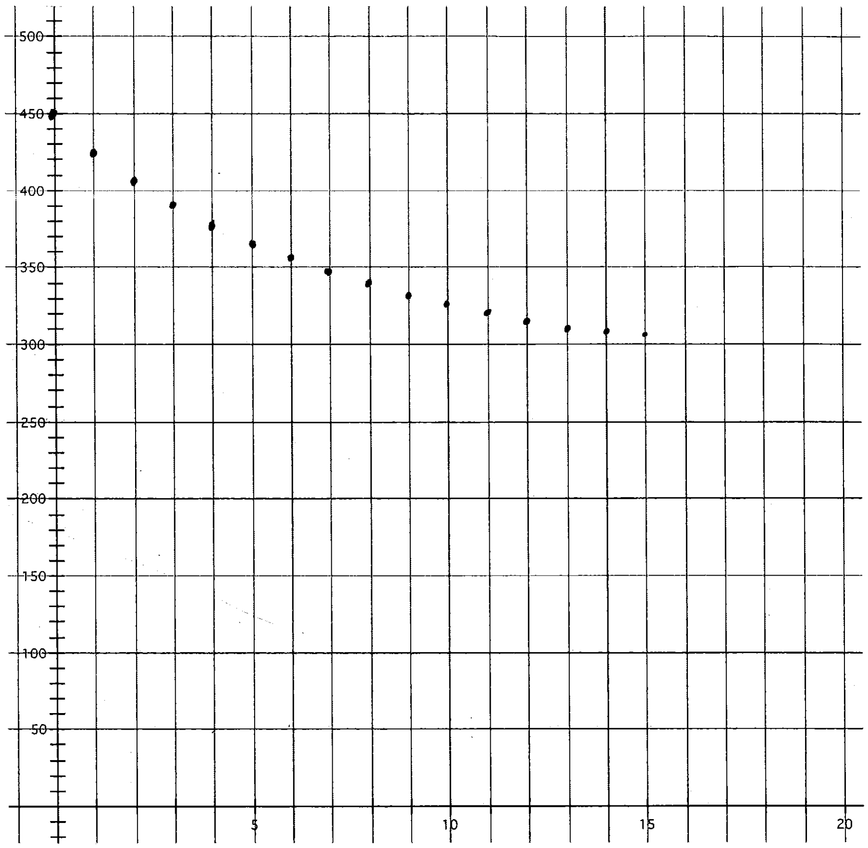
$u_0 = 450$

$u_n = (1-.15)u_{n-1} + 45 \quad n \geq 1$

b. Find the amount of chlorine after 1 day, 2 days, and 3 days:

0	450
1	427.5
2	408.375
3	392.11875

c. Create a graph that shows the chlorine level after several days. Describe what happens to level of chlorine in the long run.



$\{0, 450\}$

$\{Ans(1)+1, Ans(2) \times .85 + 45\}$

Approaches 300

d. Algebraically find the long-run value (LIMIT) of the amount of chlorine in the pool.

LIMIT IS REACHED WHEN VALUES STOP CHANGING, SO $C = u_n = u_{n-1}$

$u_n = .85_{n-1} + 45$

$C = .85C + 45$

$-.85C \quad -.85C$

$\frac{.15C}{.15} = \frac{45}{.15}$

$\boxed{C = 300}$

Example 2: Given the shifted geometric sequences below, find the long-run value (LIMIT):

a. $u_0 = 20$
$u_n = u_{n-1}(1 - 0.23) + 12$

$C = .77C + 12$

$-.77C \quad -.77C$

$\frac{.23C}{.23} = \frac{12}{.23}$

$C = 52.17391304$

b. $u_0 = 5$
$u_n = u_{n-1}(1 - 0.25) + 2$

$C = .75C + 2$

$.25C = 2$

$C = 8$